Risk Perception with Imperfect Information and Social Interactions: understanding group polarization

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Abstract

This article examines the group polarization process when agents are faced with a risk for which the probability of occurrence is not perfectly known. First, we show information destruction through an informational cascade phenomenon. Then, we analyze how this inefficiency is amplified if individuals with the same type of risk-related behaviour group together. Two extensions are detailed: consideration of the possibility that individual agents participate in more than one group; introduction of agents highly confident in their own information, enabling cascades to be 'broken'. Under conditions, the behaviour of these agents, costly at the individual level, is collectively efficient.

Keywords: Risk perception, Informational imperfection, Social interactions.

Classification JEL: D81, D83, D85.

Introduction

Decision under risk appears to be highly polarized. Firstly, polarization across different kind of risk: several risks are systematically underestimated while others are overestimated[2]. Secondly, polarization of a given risk among different groups and over time. As stressed by Kuran and Sustein (1999) individual perceptions of a risk are framed socially through interactions with others. Word of mouth is one of the important factors in risk evaluation. This is consistent with findings that perceptions and attitudes toward a given risk can vary greatly across cultures and across time (Douglas and Wildavsky 1982). Crime perception, for example, offers major variations among cities, which are not purely correlated with crime level as we can see in the International Crime Victims Surveys. While nuclear power has a relatively wide acceptance in France, it arouses fear in Germany. Such variations across cultures and across time is a widespread phenomenon that deserves research interest. The motivation of the paper is based on this point.

When agents form beliefs without possessing precise information, they rely on a small number of heuristic principles that reduce the scale of the cognitive task involved to simpler pro-

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[2] Tversky and Koehler (1994) ask to evaluate the probability for different causes of death. Homicide appears to be overestimated (evaluated at 10% whereas it represents in fact only 1% of death). Conversely, heart diseases are underestimated (22% instead of 34%).

cesses of judgement[3]. In this context, social processes contribute to the evolution of risk perception. Nevertheless, false beliefs can spread and lead to collective irrationality, despite the fact that each agent is individually rational (Hakes and Viscusi 1997). In this context, informational cascades – ie optimality for an individual to follow the behaviour of the preceding individual without regard to his own information - can produce systematic and persistent misperceptions. Individuals converge on one behaviour on the basis of some but very little information. These phenomena can explain both the conformism, polarization and the fragility of human behaviour as stressed by Bickchandani, Hirshleifer and Welch (1992), hereafter designed as BHW. Such phenomena interact with psychological dimensions, most notably emotion.

While real decisions in risky context are based on highly complex motivations, the cascade theory can nevertheless provide meaningful insights to understand localized conformity in risk-related behaviour. As cascade theory is a theory of information, it is central to understand decision under uncertainty. If someone has little information about the magnitude of a risk, it is easier to be guided by others in order to economize on both mental effort and time. Such cognitive biases can be further amplified by the role played by emotion in individual and group behaviour, as we will demonstrate below[4]. Obvisously, psychological traits have been deeply analyzed in risk perception theory. But most of them say nothing about the social dimensions that shape risk judgments and interact with psychological dimensions. In this article, we try to articulate both of these elements.

The inspiration of our model is based on the BHW paper. We analyze here specifically the case of an imperfect knowledge on the probability of occurrence. In fact, the role of social interactions in a context of imperfect information has been the subject of surprisingly little research to understand risk-related behaviour. Kuran and Sunstein (1999) stress the availability heuristic already formulated by Kahneman and Tversky (1974): a risk is more present in the mind and is considered thereby to be more plausible if it is frequently raised in discussions. More generally, the authors stress that the availability of information shapes judgments about the magnitude of various risks and consequent individual actions. One risk may gain salience and become the object of tight regulation while another, which experts deem equivalent, is treated with less regulation. If the informational cascade concept has been used to understand the insurance market[5], little work has been produced to show how personal traits and social interactions interact in the elaboration of risk perception and mitigation at the individual level. For the purpose of the present article, we propose a new approach, allowing to understand group polarization in risk evaluation and mitigation. In the first section, we analyze the information destruction process through the classic informational cascade phenomenon. The second section addresses the specific case of *homophily* – ie when individuals with the same type of risk-related behaviour group together – which amplifies information destruction. The third and fourth sections aim at studying the ways to improve informational efficiency: firstly,

[3] Kahneman and Tversky (1974) describe three broad categories of heuristics: the representativeness heuristic (when agents need to analyze a risk situation, they often do so by likening it to a set of previously known situations that appear to them to be similar); the availability heuristic (the risk is assessed on the basis of the information that comes most readily to mind, that is to say the information that is the most publicised, striking or recent); the anchoring heuristic (estimation of risk in this case is based on a previous event – the 'anchor' – which is taken as a reference and adjusted to represent the situation at issue). The well-documented availability heuristic in particular shows that agents form beliefs about risks on the basis of very little information. The heuristics they highlight are convenient, but lead to biased estimates.

[4] The recent inputs of neurobiological research have shown that emotion plays a major role in rational decision-making under uncertainty. See notably Rustichini et al. (2011).

[5] See for example d'Arcy and Oh (1997) or Seog (2008).

with agents participating in several separate groups; secondly, with agents having excessive confidence in their own information.

1 – Imprecise probabilities and belief propagation: the case of informational cascades

For expositional clarity, we begin with the classical BHW model. The probability of risk occurrence is supposed to take two values, one low - p' - and the other high - p''. The real probability of occurrence is one of these two probabilities but individual agents have an imperfect knowledge about it.

The consequence of risk occurrence can be estimated to be V (the same for each agent). The cost of a mitigation action is C (the same for each agent). We assume:

$$p'.V < C < p''.V$$

so that it is optimal to adopt a mitigation action if and only if the probability is high. Each individual formulates successively a belief as to p' or p'' and adopts behaviour that is rational on the basis of that belief (mitigation if the economic agent thinks that the probability is high and no mitigation in if the probability is thought to be low). The ordering of individuals is exogenous and is known to all. The ex-ante probability that $p = p'$ is one-half. Each agent receives private information or a signal before decision. This private information can take the value $S = H$ or $S = L$. It can be interpreted as a signal deduced from personal experience or personal information sources regarding the risk. In addition, each of them, with the exception of the first, can observe the previous decisions (for example, leaving home with an umbrella reveals that you think it will rain). The information conveyed by actions is the most credible about beliefs since everybody must reveal them without lying and sometimes without speaking (as in the umbrella example). But, the private information of earlier agents can't be observed. Based on private information and observations he decides whether or not to adopt mitigation action.

Without loss of generality, we assume that the real value of the probability is p'. Thus, it is optimal not to adopt any action of risk mitigation. Because of the imperfect information, one portion of the total population will assess the probability correctly, while another portion will overestimate the risk and adopt inappropriate mitigation action. The opposit case, which leads to an underestimation of the risks, is symmetrical and our model does not therefore reduce the generality of the question.

The uncertainty structure of the model can be presented as follows. The prior belief is $P(p') = P(p'') = 1/2$. The private signals are iid among individuals, conditional of the state of the nature (here p' without loss of generality) and this structure is common knowledge. $P(L/p') = P(H/p'') = q \geq 1/2$. By difference $P(L/p'') = P(H/p') = 1-q$. We can call q the "strength" of the signal or the informativity. By Bayes rules, we have: $P(p'/L) = P(p''/H) = q$ and $P(p'/H) = P(p''/L) = 1-q$.

Let us investigate the possibility of a cascade. As a tie-breaking convention, an individual who can infer as many signals in the opposit directions behaves in accordance with his own signal. Thus, the first individual adopts mitigation action if the personal signal is *H* (the underlying belief is a probability p'') and rejects this action if the personal signal is *L* (the underlying belief is a probability p'). If the first individual adopted, the second one adopts if his signal is also *H.* However, in case of *L,* he rejects (he can infer the same number of signals and behaves according to his own information). Similarly, if the first had rejected, the second rejects in case of signal *L* and adopts in case of *H*. The third individual has three possible situations. Both have adopted the mitigation measure before him and he adopts (a cascade based on a p'' belief occurs); both have rejected the mitigation measure and he rejects (a cascade based on a p' belief occurs); one has adopted and the other rejects the mitigation measure. In the last case *H* and *L* cancel each other and the third individual is in the same situation as the first one. His personal signal determines his choice. A similar analysis shows that the fourth individual would be in the same situation as the second, the fifth as the third, and so forth…Given that p' is the correct value of the probability, with this decision rule, we can derive the probability of correct/incorrect/no cascade after two individuals:

- No cascade occurs if there is one *H* and one *L,* with a probability $2.q.(1-q)$
- A correct cascade if there are two *L* signals, with a probability q^2
- An incorrect cascade if there are two *H* signals, with a probability $(1-q)^2$

More generally, two consecutives *L* after an alternance of signals initiates a correct cascade based on a *p'* perception and two consecutives *H* after an alternance of signals initiates an incorrect cascade based on a *p''* perception. The last case in which no cascade occurs is *H* and *L* alternance.

The probability that there is no cascade after an even number of *n* agents (NC) is therefore:

$P_{nc} = (2.q.(1-q))^{\frac{n}{2}}$ which tends to 0 as *n* tends to infinity (we consider the case in which the number of individuals in the group is very large).

The probability of a correct cascade (CC) is obtained by summing the probabilities that it will appear after each agent, up to agent *n.* This can be written as follows:

$$P_{cc} = \sum_{k=1}^{n/2} q^2 \left(2q(1-q)\right)^{k-1} = \frac{q^2\left(1-\left(2q(1-q)\right)^{n/2}\right)}{1-2q(1-q)}$$, which tends to :

$$f_q = \frac{q^2}{q^2+(1-q)^2}$$

Similarly for the probability of a cascade that is incorrect (IC):

$$P_{ic} = \sum_{k=1}^{n/2} (1-q)^2 \left(2q(1-q)\right)^{k-1} = \frac{(1-q)^2 \left(1-\left(2q(1-q)\right)^{n/2}\right)}{1-2q(1-q)}$$, which tends to:

$$1 - f_q = \frac{(1-q)^2}{q^2 + (1-q)^2}$$

If the population includes a large number of individuals and social groups (which supposes that the society is socially fragmented), according to the law of large numbers, a portion f_q of the total population will correctly assess the probability of occurrence p and not undertake mitigation action (supposing that the real probability is p'), whereas a complementary portion $1 - f_q$ will arrive at a biased assessment and undertake mitigation action whereas it is not necessary. *Local informational cascades* can be limited, for example, to a geographical area, a demographic subgroup, etc… Such a model proposes an explanation of the widely accepted fact of great variability and polarization among different groups – cities for example – in risk perception and regulation (the case of crime is highly relevant to illustrate this issues). That is why this framework provides so meaningful insights in risk perception understanding.

The decision rule presented above is rational at individual level, but leads to the destruction of information. Indeed, cascade prevents the aggregation of information on risk of numerous individuals. If the information of many previous individuals is aggregated, later individuals should converge to correct mitigation decision. However, once a cascade has started, actions convey no information about private signals and an individual's action does not improve later decisions. If reporting the occurrence of a given risk can be seen as an action, the perception of a given risk can be heightened by giving high publicity and report to existing occurrences whatever personal information. In this case, the collective misperception (high probability cascade in case of low probability risk) would make agents more willing to report occurrences, reinforcing high probability cascade. By the same logic, the perception of another risk can be lowered through information suppression of given occurrences. In this case, the collective misperception (low probability cascade in case of high probability risk) would make agents less willing to report occurrences, reinforcing low probability cascade.

If the aim of our paper is to understand the polarization process and local conformity in terms of risk perception, we must also analyze how the cognitive issue raised above can be amplified by emotional factors. As we know, emotional factors are of tremendous importance in risk related behaviour. Indeed, some individuals are worry while others are abnormally unconcerned when faced with certain types of risk. Moreover, such agents often tend to form groups, and this has the effect of increasing perceptual bias in a systematic – i.e. non-erratic – fashion. In the case considered here – a low value for p – worriers will amplify their propensity to overestimate risk. Similarly, relaxed personalities will show a tendency to underestimate risk when the probability is high. We can now examine the formal processes underlying such group polarization.

2 – Information destruction: the case of homophily in groups

We know, in many cases, that the constitution of groups is based on partners with similar characteristics. For example, politically engaged newspapers are most of time read by a part of population of the same opinion. It is partly due to this particular feature that ideas, true or false, spread most effectively. This is referred to as *homophily* (See most notably McPherson, Smith-Lovin and Cook 2001). In the case studied here, we consider that groups form between partners who have a same bias towards a given risk (worriers and worry-free). Here again, examples can be found. The green party will include more individuals for which environmental risks are major issues and, among them, bad news conveying probability are frequently overweighted. The right wing will include more individuals for which crime constitutes a major risk and bad news conveying probability are also frequently overweighted. Such kind of polarization process can be called *ex ante polarization.* They are due to opposit risk attitudes. And homophily is crucial in risk related behaviour. In fact, people's preconceptions about risk can exhibit selective trust and mistrust about signals. This erroneous probability perception is not common knowledge. The structure can be modelled as follows:

- The real strength of the signal is q as we have $P(L/p') = P(H/p'') = q \geq 1/2$ and $P(p'/L) = P(p''/H) = q \geq 1/2$. But, worriers will perceive a signal of high probability risk as having a strength of *q+x;* a signal of low probability risk as having a strength of *q-x*. We can note *Q* this probability perception.
- In this new framework, $Q(p'/L) = q - x < q = P(p'/L)$, with $q - x > \frac{1}{2}$, and $Q(p''/H) = q + x > q = P(p''/H)$. By difference, $Q(p'/H) = 1 - q - x < 1 - q = P(p'/H)$ and $Q(p''/L) = 1 - q + x > 1 - q = P(p''/L)$. These agents thus attach more importance to pessimistic signals.
- The worry-free will perceive a high probability signal as having a strength of *q-x;* a low probability signal will have a perceived strength of *q+x*. We can note *R* this probability perception. In this new framework, $R(p'/L) = q + x > q = P(p'/L)$ and $R(p''/H) = q - x < q = P(p''/H)$, with $q - x > \frac{1}{2}$. By difference, $R(p'/H) = 1 - q + x > 1 - q = P(p'/H)$ and $R(p''/L) = 1 - q - x < 1 - q = P(p''/L)$. They thus give greater weight to optimistic signals. Such new tie-breaking assumptions change the behaviour of the decider only when the two kinds of signal are of equal number. They change the probability of correct and incorrect cascades. Some definition first.

Definition : Mean Preserving Group Polarization (MPGP) is defined as a situation where the groups presenting the two types of bias – groups of worriers and worry-free – are equally represented within the population. Thus, the mean perception of the signal is keeped unchanged.

Proposition 1 : In case of MPGP, the share of the population estimating correctly p is lower than without *homophily*. This means that the *ex ante polarization* process itself brings information destruction at the collective level.

Proof : In principle, four cases may arise: high probability risk and a group of worriers; high probability risk and a group of worry-free agents; low probability risk and a group of worriers; low probability risk and a group of worry-free agents. The first and fourth cases are in fact symmetrical and we have chosen, without loss of generality, a real probability that is low. The two types of groups are matched with two distinct probabilities f_{q1} and f_{q2} of obtaining correct cascades with respectively worry-free and worriers. The first case is referred to here as 'positive homophily' (PH) insofar as the homophily is oriented in the direction of reality (bias of worry-free agents toward perception of low probability while the probability is really low). In the second case, the term used is 'negative homophily' (NH), since the homophily runs counter to the facts.

First, the case of worry-free agents. Let us investigate the possibility of a cascade. The first individual adopts mitigation action if the personal signal is *H* as $R(p''/H) = q - x > \frac{1}{2}$ (the underlying belief is a probability p'') and rejects this action if the personal signal is *L* as $R(p'/L) = q + x > \frac{1}{2}$ (the underlying belief is a probability p'). If the first individual rejected, the second one rejects in any case. If its signal is *L,* like in the first cascade framework. But also, in case of signal *H,* since the signal corresponding to a low probability is perceived as stronger than the contrary signal. There will be therefore a correct cascade in any case. If the first had adopted the mitigation action, the second rejects in case of signal *L* and adopts in case of *H*, thus triggering an incorrect cascade. The third individual has three possible situations. Both have adopted the mitigation measure before him and he adopts (a cascade based on a p'' belief occurs); both have rejected the mitigation measure and he rejects (a cascade based on a p' belief occurs); one has adopted and the other rejects the mitigation measure. In the last case, the third individual is in the same situation as the first one and his personal signal determines his choice as the parameter *x* is supposed to be low and serves only as a tie-breaking convention in case of an equal number of each signal. A similar analysis shows that the fourth individual would be in the same situation as the second, the fifth as the third, and so forth…Given that p' is the correct value of the probability, with this decision rule, we can derive the probability of correct/incorrect/no cascade after two individuals:

- No cascade occurs if there is first one *H* and then one *L,* with a probability $q(1-q)$
- A correct cascade if there is first one *L* signal, with a probability q
- An incorrect cascade if there are two *H* signals, with a probability $(1-q)^2$

The probability that there is no cascade after an even number of *n* agents (NC) is therefore:

$P_{1nc} = (q(1-q))^{\frac{n}{2}}$ which tends to 0 as *n* tends to infinity (we consider the case in which the number of individuals in the group is very large). It corresponds to an alternance of *H* and *L* in this order.

To observe a correct cascade, it is necessary and sufficient that a signal *L* to be received by the first agent, or by the third (after an alternance of *H* and *L* in this order), or by the fifth (after an alternance of *H* and *L* in this order), and so on. Thus, the probability of there being a correct cascade can be written as follows:

$$P_{1cc} = \sum_{k=1}^{n/2} q\left(q\left(1-q\right)\right)^{k-1} = \frac{q\left(1-\left(q\left(1-q\right)\right)^{n/2}\right)}{1-q\left(1-q\right)}$$

It is then easy to see that:

$P_{1cc} \xrightarrow{n\to+\infty} f_{q1}$, where

$f_{q1} = \frac{q}{\left(1-q+q^2\right)}$. Moreover, the following relation is easily verified:

$$f_{q1} = \frac{q}{\left(1-q+q^2\right)} \geq f_q = \frac{q^2}{q^2+\left(1-q\right)^2}$$

If the perceptual bias of the agents in the group tends toward attributing greater strength to the informative signal (a low real probability and a group of non-worriers), the probability of a correct cascade is higher than it is for rational agents. The complementary probability of an incorrect cascade, at the limit can be deduced from the above calculation:

$$1-f_{q1} = \frac{\left(1-q\right)^2}{\left(1-q+q^2\right)} \leq 1-f_q .$$

In the case of a group of worriers, we find in the same way, at the limit:

$f_{q2} = \frac{q^2}{\left(1-q+q^2\right)}$. In addition, the following relation is easily verified:

$$f_{q2} = \frac{q^2}{\left(1-q+q^2\right)} \leq f_q = \frac{q^2}{q^2+\left(1-q\right)^2}$$

The complementary probability of an incorrect cascade can then be deduced in the following form:

$$1-f_{q2} = \frac{1-q}{\left(1-q+q^2\right)} \geq 1-f_q$$

If the perceptual bias of the agents in the group will tend to attribute less strength to the informative signal (a low real probability and a group of worriers), the probability of a correct cascade is lower than for rational agents. We can now see how the probabilities of giving a correct (incorrect) knowledge about the risk evolve as a function of the parameter *q*.

C(I)CWH: a correct (incorrect) cascade without homophily.

C(I)CPH: a correct (incorrect) cascade with positive homophily. This is the case in which there is a grouping of worry-free individuals (and the probability is low).

C(I)CNH: a correct (incorrect) cascade with negative homophily. This is the case in which there is a grouping of worriers (and the probability is low).

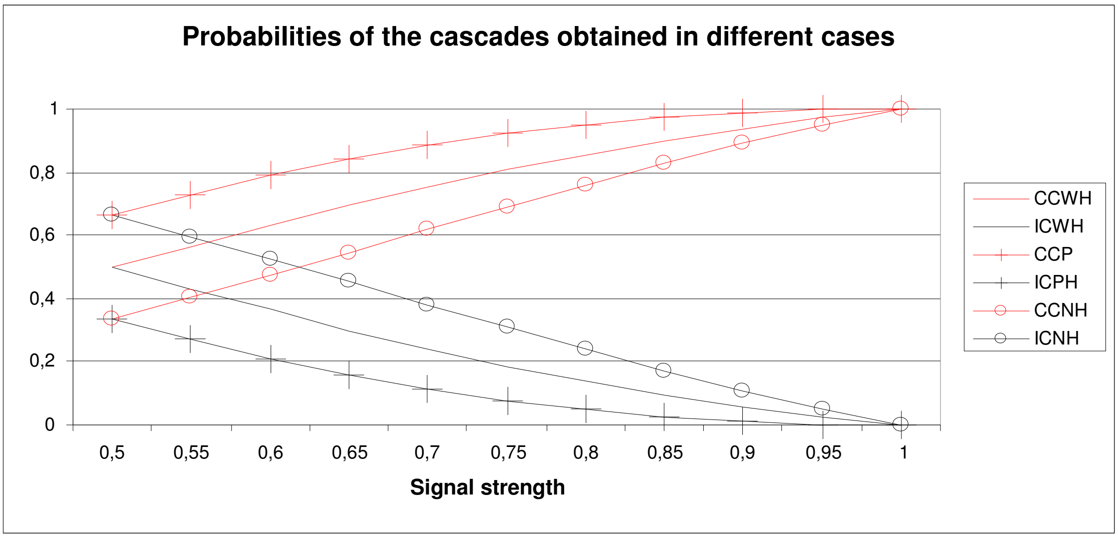

Figure 1 : Proportion of correct and incorrect cascades in the presence of homophily

What happens in the case of a MPGP? If there are as many groups of worriers as groups of non-worriers and a large population, we can analyze the specific effect of group polarization and write the share of the population estimating correctly the risk:

$$f_{q3} = \frac{1}{2}.f_{q1} + \frac{1}{2}.f_{q2} = \frac{q+q^2}{2.\left(1-q+q^2\right)}.$$

It is then easy to show that for any value of q included in the interval $\left[\frac{1}{2},1\right]$:

$$f_{q3} = \frac{q+q^2}{2.\left(1-q+q^2\right)} \leq \frac{q^2}{q^2+\left(1-q\right)^2} = f_q \,.$$

This means that even if the two types of groups are equally represented within the population, the process of polarization of the groups will itself bring information destruction at the collective level.

□

Thus, we have emphasised the role of personal bias for decision under risk in a cascade framework. Some individuals, those that we have called ‘worriers’ here, attribute greater weight to worrying signals, others, those described as ‘non-worriers’, attribute greater weight to signals that are reassuring. But, the intuition behind the proposition stated above is that the process of *ex ante polarization*, due to *homophily,* is by itself information destructive even if the two biases are equally represented within the population. This *ex ante polarization* may explain why whole segments of the population overestimate or underestimate certain classes of risk with, on average, a destruction of information at the collective level.

We can now look at how these crucial polarization traits can be modified. For this, we go on consider two types of organisation based on two types of individual and analyze the degree to which those individuals can provide information beneficial to the community.

3 – Avoiding information destruction: nomadism

We know that communities contain subcommunities mostly based on "inside interactions". In case of *homophily,* a given subcommunity contains only members of the same kind of risk perception: Bayesian, worry-free or worrier. In this context, if a cascade occurs in a subcommunity, its members can ignore information provided by outsiders and only follow the cascade. But, it is reductionist to consider that individuals will belong necessarily to only one group whose behaviour they follow slavishly. In this section, we analyze the case of agents with multiple memberships of different groups. We define a "Nomad" as someone who observes more than one group before to take a decision. Nomads do not see risk in the same way as others and nomadism as well as openness of networks influence greatly risk related behaviour[6]. They can play a pivotal role, able to store up information from different sources. Their decisions – or information release - can convey new information for the benefit of the community as a whole. We do not modelize here the process by which information can be passed to the community and focalise only on the information acquisition process by nomads. Three assumptions can be made:

- The observations by the "Nomad" are made after that cascades have occurred (the probability that there is no cascade decreases rapidly as *n* increases and the number of individuals in each group is assumed to be very large).
- The choice of the observed groups by each "Nomad" is assumed to be randomly done inside one of the four following communities: between worry-free subcommunities, Bayesians, worriers or a mix between worry-free and worriers (MPGP situation, ie a community where the two kind of bias are equally represented).
- We assume that the groups in which the "Nomads" participe are not interconnected so that the decision made by the "Nomad" does not modify the existing cascades.

Let us consider an individual who belongs to several different groups (*m* groups). The agent possesses more information due to his comparison of information from the different groups in which he is a participant. Thus, he will form his belief in accordance with the majority of the cascades he has observed and will adopt behaviour aligned with that belief. If there are the same numbers of behaviour patterns of the two types, the individual will follow his own sig-

[6] For example, the type of involvment in interpersonal networks governs the perception of crime. Individuals who have links with only one closed network (comprising, within the same circle, friends, family, colleagues, leisure activities, and so on) evidence a greater tendency to overestimate this risk. Conversely, the openness of networks modifies the reaction to the risk through the availability of more information.

nal, which is correct with a probability q. We still suppose, without loss of generality, that the correct probability is low.

Proposition 2 : Let us suppose that the "Nomad" observes groups in one of the three following communities: worry-free, bayesian agents or a MPGP situation. Then, the probability of correct belief about p increases with m and tends to 1 when m tends to infinity. If a "Nomad" observes groups of worriers, the probability of a correct belief about p increases with m and tends to 1 when m tends to infinity if and only if:

$$q > \left(\sqrt{5} - 1\right)/2 .$$

Proof : With Bayesian agents, ie without *homophily,* behaviour can be captured formally using a binomial law with parameters f_q and m. The probability that the number of correct cascades is r can be written as follows:

$$\sigma(r) = \frac{m!}{r!(m-r)!} . f_q^{\,r} \left(1 - f_q\right)^{m-r}$$

The probability of an equal number of cascades of each type can take two values. If m is an even number, its value is:

$$\sigma\left(\frac{m}{2}\right) = \frac{m!}{\frac{m}{2}!\frac{m}{2}!} . f_q^{\frac{m}{2}} \left(1 - f_q\right)^{\frac{m}{2}}$$

If m is an odd number, its value is 0.

The probability that an individual, having observed m cascades, grasp correctly the probability of occurrence for the risk can then be written:

If m is even:

$$g(q,m) = \sum_{r=\frac{m}{2}+1}^{m} \frac{m!}{r!(m-r)!} . f_q^{\,r} \left(1 - f_q\right)^{m-r} + q.\sigma\left(\frac{m}{2}\right)$$

If m is odd:

$$g(q,m) = \sum_{r=\frac{n+1}{2}}^{m} \frac{m!}{r!(m-r)!} . f_q^{\,r} \left(1 - f_q\right)^{m-r}$$

We can now look at how the function g evolves with the strength of the signal and the number of observed groups:

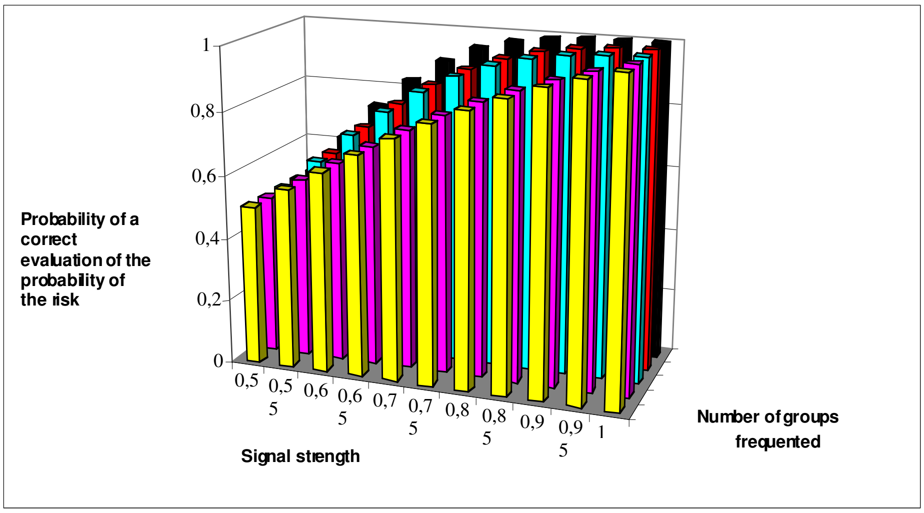

Figure 2 : Variation of g with the signal strength and the number of groups

According to the law of large numbers, the empirical probability (the proportion of correct cascades) tends to the theoretical probability (the probability that any given cascade is a correct one) as the number of observations rises. If f_q is greater than ½, we can express formally:

$$\frac{r}{m} \xrightarrow{m \to +\infty} f_q > \frac{1}{2}$$

We can obtain from this:

$$\forall f_q > \frac{1}{2}, \varepsilon, \mu > 0, \exists m_0$$

such that : $\forall m \geq m_0$, then:

$$P\left(\frac{r}{m} \in \left] f_q - \mu, f_q + \mu \right[\right) \geq 1 - \varepsilon$$

which shows us, if we choose $\mu < f_q - \frac{1}{2}$:

$\forall \varepsilon > 0, \exists m_0$ such that, $\forall m \geq m_0$, then:

$$P\left(\frac{r}{m} > \frac{1}{2}\right) \geq 1 - \varepsilon$$

In this case we do in fact arrive at:

$$g(q,m) \xrightarrow{m \to +\infty} 1$$

If the Nomad observes groups in the worry-free or MPGP communities, the same can apply because f_{q1} and f_{q3} are also superior to ½. The last case to be considered is the case of worriers, for which the probability of correct cascade is f_{q2} . The condition can be written:

$$f_{q2} = \frac{q^2}{\left(1 - q + q^2\right)} > 1/2 \Leftrightarrow q > \left(\sqrt{5} - 1\right)/2$$

If this latter condition is not verified, the binamial law is based on a probability lower than ½ and, according to the law of large numbers:

$$g(q,m) \xrightarrow{m \to +\infty} 0 \; .$$

□

Obviously, as there is not loss of generality, the same kind of result hold in case of high probability of occurrence. In this latter case, if a nomad observes groups in worriers, Bayesians or MPGP communities, he will gain a better idea of a given risk. In case of observations made in groups of worry-free agents, a better idea is gained under the same conditions as above.

Therefore, participation in more than one group enables most of time an individual to gain a better idea of the risk and allows better mitigation strategies. For example, it is possible to understand the system of separation of powers (checks and balances in US) as a kind of protection against informational cascades about given subjects. This observation, when analysing behaviour in an uncertain universe, concords with many studies into the utility of 'weak ties' that allow individuals to draw additional information from their social environment (See notably Granovetter 1973). But, if the signal is not very informative and that the environment is polarized by emotion contrary to the reality of the risk, "nomadism" can on the contrary reinforce the process of information destruction. In this context, it is desirable, but only under conditions, that such kind of agent should pass on the information obtained by comparison with other sources in a public manner. This would presuppose to take a public signal into consideration in a new model. Thus, *if several conditions are met*, even if establishing links is costly at individual level, the collectivity can benefit from such agents. Internet, for example, might constitute a crucial tool in this context. More generally, diffusion of risk information, after compiling wide range of risk levels and probabilities could be of great help.

Other structures may be envisaged as channels to improve information efficiency. Once again, they are based on types of agents with certain specific characteristics.

4 – Avoiding information destruction: resiliency to conformity

Overconfidence and resiliency to conformity constitutes a second way to improve information efficiency. The individuals we identify here as "overconfident" are agents who place more weight on their own information than Bayesian individuals. According to DeBondt and Thaler (1995) "Perhaps the most robust finding in psychology of judgement is that people are over-confident". Such overconfidence induces individuals to undertake actions that more rational individuals might not undertake. Particularly, overconfidence is central to understand risk related behaviour when agents are uncertain about the probability of occurrence.

Their behaviour appears suboptimal at the individual level, but can enable cascades to be broken and, consequently, may provide new information at the collective level. A non-zero proportion of such agents might in this way turn out to be optimal at the collective level if the aim is to grasp the real probability of occurrence of a given risk and put in place accurate mitigation processes. We suppose here that the majority of agents behave in accordance with the model we have described above. If the latter were the only category of individual, once a cascade was set up it would no longer be possible to infer agents' personal signals since their behaviour will not be dependent on it. Conversely, "overconfident" individuals are more sceptical about outside information and more enthusiastic about information that is internal – their own. The presence of such overconfident individuals who act on their own information and can ignore the actions of others in the group has been demonstrated in laboratory settings by Anderson and Holt (1996). How is risk related behaviour, both individual and collective, modified by this phenomenon?

Overconfidence can be captured formally as follows. The process unfolds as if the "overconfident" agents believed – wrongly – that the accuracy of their own signal is $q' > q$. We have seen in the general case that it is sufficient for the first two individuals to adopt the same belief for all those who follow to adopt it in turn. In this new context, we suppose that *k* individuals are required (where $k \geq 2$) to convince these excessively confident agents. *k* and – *k* are said to be 'absorbing states'. Obviously, the value of *k* depends directly from *q'*. A distinction can be made between two extreme cases:

- If $q' = q$, then all the individuals will behave in a Bayesian fashion, where *k=2*.
- If $q' = 1$, then the state is said to be 'critical' and $k = +\infty$.

Proposition 3 : If k is designed as the level of overconfidence, the probability of correct belief about *p*, whatever the proportion of overconfident agents, can be written[7] :

$$f_q(k) = \frac{q^k}{q^k + (1-q)^k}$$

Proof : Let us denote as D_n the difference between the numbers of 'low probability' and 'high probability' signals that can be inferred after *n* individuals, while keeping in mind that the actual probability is low (the opposit case can be dealt with symmetrically). Considering, as above, that each of the groups has a very large number of individuals, we can calculate the

[7] This result is not modified in case of *homophily* between groups. Indeed, in case of *homophily,* only the tie-breaking assumptions are modified by a change in the perception of the signal *q* (perceived as *q+x* or *q-x*). But, the value of *x* is assumed to be very low and produces a change in decisions only if the number of opposit signals are equal. In other words, we assume *x<<q'- q,* which does not modify the result below.

probability of a correct (or, respectively, incorrect) cascade. Even if confident agents are present in very small proportion, there will be no cascade so long as $|D_n| < k$. We suppose again that there is a very large number of agents, such as to ensure that there is always a cascade.

The probability of observing a correct (or, respectively, incorrect) cascade can be calculated in the same way as for the gambler's ruin problem. The process unfolds as if the gambler were in possession of *k* euros at the start, with a probability *q* (*1-q* respectively) of winning (or losing, respectively) one additional euro at each bet. Will he reach *2.k* euros first or be ruined? The first case corresponds to a correct cascade, the second to an incorrect cascade.

We know that the probability he will win is a weighted sum of the probabilities of a win in the bets that follow, which is written:

$$P(2.k / k) = q.P(2.k / k+1) + (1-q).P(2.k / k-1)$$

The characteristic polynomial of this recurrence relation is written:

$x^2 - \frac{x}{q} + \frac{1-q}{q} = 0$, which has *1* and $\frac{1-q}{q}$ as its roots, and which gives:

$$P(2.k / k) = A + B.\left(\frac{1-q}{q}\right)^k$$

The 'boundary' conditions give:

$$A + B = 0$$
$$A + B.\left(\frac{1-q}{q}\right)^{2.k} = 1$$

and we then find:

$$P(2.k / k) = \frac{1}{1+\left(\frac{1-q}{q}\right)^k} = \frac{q^k}{q^k + (1-q)^k} = f_q(k)$$

□

In the same way, the – complementary – probability that there will be an incorrect cascade (possibly after a very large number of agents if there are only a few individuals with exaggerated confidence in their own information) is written:

$$1 - f_q(k) = \frac{(1-q)^k}{q^k + (1-q)^k}.$$

And we do find once again in the Bayesian case the proportions seen above:

$$f_q = \frac{q^2}{q^2 + (1-q)^2}$$

$$1 - f_q = \frac{(1-q)^2}{q^2 + (1-q)^2}$$

We can see below the variations of the proportions established with the parameters q (signal strength) and k (degree of confidence in their information felt by so-called 'confident' individuals).

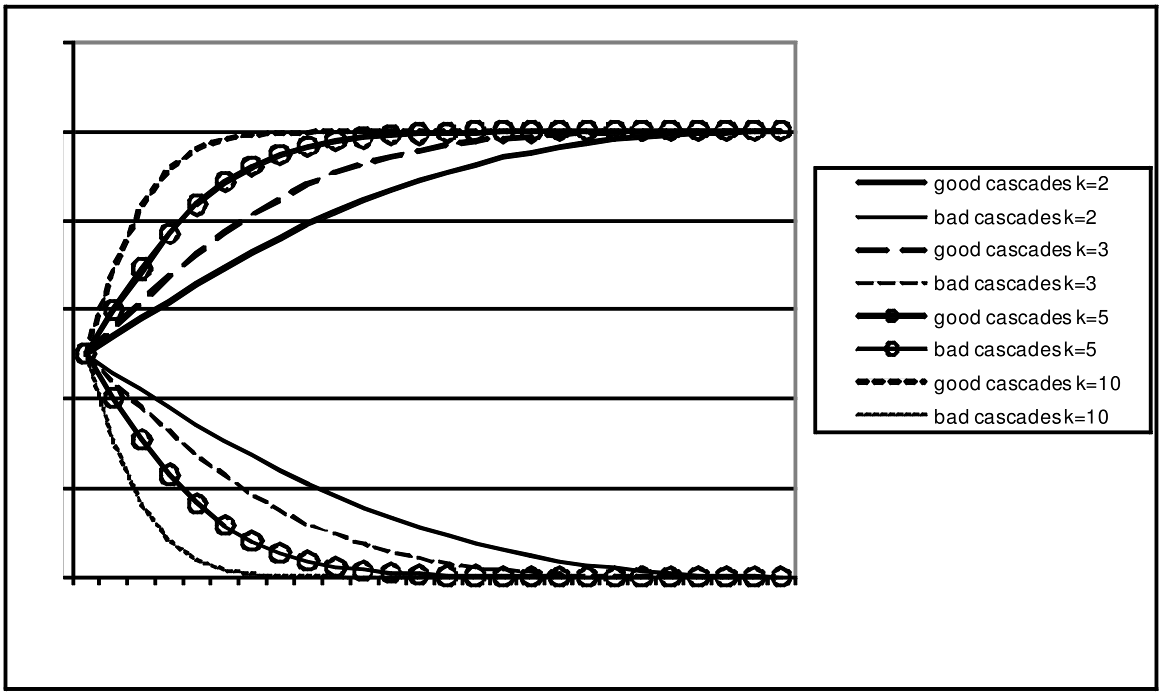

Figure 3 : Proportions of correct/incorrect cascades in the presence of overconfidence

We can see here that individuals who present excess of confidence in their own information can limit collective inefficiency in addressing risk. By ignoring informational cascade, the actions of overconfident individuals convey their private information. Obviously, the degree of confidence is of prime importance for the capacity of such agents to withstand cascades. Nevertheless, when the groups of individuals are very large, even a tiny proportion of such individuals will enable the formulation of more accurate judgements as to the probability of occurrence of a risk. We must bear in mind that this type of behaviour is efficient at the collective level (as it allows to adopt better mitigation actions), but costly for each of these individuals, who do not evaluate the probability of occurrence optimally. Thus, they adopt unsuitable (too weak or too strong) protective measures. For example, a low level in fear of crime can enable overconfident agents to go outside despite the danger while restoring confidence in the neighbourhood. Therefore, it may be possible to envisage compensating such agents – by an insurance scheme, for example - while having them into groups within which the perception of probability is an obstacle to the implementation of efficient safety policy.

Conclusion

The modelling carried out above tends to analyze a characteristic of inefficiency in the sharing of information, which supplements both the research on risk evaluation by agents and the economics of social interactions. Group polarization and localized conformity can be amplified when similar individuals group together (*ex ante polarization*). We then considered some additional characteristics capable of improving informational efficiency. Firstly, considering the case of individuals – "Nomads" - using more than one source of information. Under conditions, Nomads have more chances of accessing correct information. We examined next the impact of the presence of overconfident agents able to 'break' conventional cascades by providing additional information to the community. Obviously, while such models give us a better understanding of the intricacies of risk perception, they do not provide us with a 'turn-key' solution for evaluation of the informational biases highlighted. Several avenues for future researches can be evoked.

Firstly, it seems important to conceive more complex interconnected groups with the possibility of strong and weak links as well as differential distances between agents. Secondly, it would be of interest to understand – in such kind of interconnected groups – how the specific agents analyzed in this paper should pass on the information obtained by comparison with other sources in a public manner. More generally, we should analyze the effects of such agents on cascades if they grasp information before that cascades have occurred. For this, a public or private signal – through Internet, for example – could be taken into consideration in a new model. Thirdly, it would be of interest to understand the determinants of 'worry' and 'worry-free' attitudes to risk depicted in section 2. Does the age, the education, the wealth… of the agents, for example, have any real influence on the way in which they perceive risk?

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